Unit 6 - Trig. Identities

Day #1 - Basic Trig Identities

Objective - You will identify and use reciprocal, quotient, pythagorean, and opposite ∡ Identities.

Identity - a statement of equality between two expressions that is true for all values of the variable(s) for which the expressions are defined.

Reciprocal Trig Identities

$\csc\theta = \frac{1}{\sin\theta}$ $\quad \sin\theta = \frac{1}{\csc\theta}$

$\sec\theta = \frac{1}{\cos\theta}$ $\quad \cos\theta = \frac{1}{\sec\theta}$

$\cot\theta = \frac{1}{\tan\theta}$ $\quad \tan\theta = \frac{1}{\cot\theta}$

Quotient Identities

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\cot\theta = \frac{\cos\theta}{\sin\theta}$

Pythagorean Identities

$\sin^2\theta + \cos^2\theta = 1$ → $\cos^2\theta = 1 - \sin^2\theta$, $\sin^2\theta = 1 - \cos^2\theta$

$1 + \cot^2\theta = \csc^2\theta$ →

$\tan^2\theta + 1 = \sec^2\theta$

Examples: Simplify

1) $\frac{\sec\theta}{\tan\theta}$

$\frac{1}{\cos\theta} \div \frac{\sin\theta}{\cos\theta}$

$\frac{1}{\cos\theta} \cdot \frac{\cos\theta}{\sin\theta}$

$= \frac{1}{\sin\theta} = \boxed{\csc\theta}$

2) $\frac{\cot\theta}{\cos\theta}$

$= \frac{\frac{\cos\theta}{\sin\theta}}{\frac{\cos\theta}{1}}$

$= \frac{1}{\sin\theta} \cdot \frac{\cos\theta}{\cos\theta}$

$= \frac{1}{\sin\theta} = \boxed{\csc\theta}$

3) $\sin X \cos X \sec X \cot X$

$\frac{\sin X}{1} \cdot \frac{\cos X}{1} \cdot \frac{1}{\cos X} \quad \frac{\cos X}{\sin X}$

$= \boxed{\cos X}$

4) $\frac{\cos X}{\sec X - \tan X}$

$= \frac{\cos X}{\frac{1}{\cos X} - \frac{\sin X}{\cos X}}$

$= \frac{\cos X}{\frac{1-\sin X}{\cos X}}$

$= \frac{\cos X}{1} \cdot \frac{\cos X}{1-\sin X}$

$= \frac{\cos^2 X}{1-\sin X}$

$= \frac{1-\sin^2 X}{1-\sin X} = \boxed{1-\sin X}$

5) $\cos^4\theta + 2\cos^2\theta \sin^2\theta + \sin^4\theta$

$= (\cos^2\theta + \sin^2\theta)(\cos^2\theta + \sin^2\theta)$

$= 1 \cdot 1$

$= \boxed{1}$

6) $(\cos\theta - 1)(\cot\theta + \csc\theta)$

$= (\cos\theta - 1)\left(\frac{\cos\theta}{\sin\theta} + \frac{1}{\sin\theta}\right)$

$= (\cos\theta - 1)\left(\frac{\cos\theta + 1}{\sin\theta}\right)$

$= \frac{\cos^2\theta - 1}{\sin\theta} = \frac{\sin^2\theta}{\sin\theta} = \boxed{\sin\theta}$

Hitung:

1) $(\sin X + \cos X)^2 + (\sin X - \cos X)^2$

2) $\cos X \tan X + \sin X \cot X$

3) $(1 + \cos\theta)(\csc\theta - \cot\theta)$

Unit 6 Day #2 – Verifying Trig Identities

Objective – You will use the basic trig identities to verify other identities.

Showing that both sides of a trig. equation are equal without solving.

* You can only work with one side at a time. (draw a vertical line separating the two sides)
* Start with the complicated side and simplify.
* Rewrite everything in terms of sine and cosine.
* Watch for the Pythagorean Identities.

Examples

Verify the following.

1) $\sec\theta - \tan\theta = \frac{1-\sin\theta}{\cos\theta}$

$\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}$

$\frac{1-\sin\theta}{\cos\theta} = \frac{1-\sin\theta}{\cos\theta}$ ✓

2) $\sec^2\theta - \tan\theta\cot\theta = \tan^2\theta$

$\frac{1}{\cos^2\theta} - \frac{\sin\theta}{\cos\theta}\cdot\frac{\cos\theta}{\sin\theta}$

$\frac{1}{\cos^2\theta} - \frac{1}{1}$

$\frac{1}{\cos^2\theta} - \frac{\cos^2\theta}{\cos^2\theta}$

$\frac{1-\cos^2\theta}{\cos^2\theta} = \frac{\sin^2\theta}{\cos^2\theta}$

$\tan^2\theta = \tan^2\theta$ ✓

3) $\frac{\sin A}{\csc A} + \frac{\cos A}{\sec A} = \csc^2 A - \cot^2 A$

$\frac{\sin A}{\frac{1}{\sin A}} + \frac{\cos A}{\frac{1}{\cos A}} = \frac{1}{\sin^2 A} - \frac{\cos^2 A}{\sin^2 A}$

$\frac{\sin A}{1} \cdot \frac{\sin A}{1} + \frac{\cos A}{1} \cdot \frac{\cos A}{1} = \frac{1 - \cos^2 A}{\sin^2 A}$

$\sin^2 A + \cos^2 A = \frac{\sin^2 A}{\sin^2 A}$

$1 = 1$ ✓

4) $\sec X \csc X = \tan X + \cot X$

$\frac{1}{\cos X} \cdot \frac{1}{\sin X} = \frac{\sin X}{\cos X} + \frac{\cos X}{\sin X}$

$\frac{1}{\cos X \sin X} = \frac{\sin^2 X}{\cos X \sin X} + \frac{\cos^2 X}{\cos X \sin X}$

$\frac{\sin^2 X + \cos^2 X}{\cos X \sin X}$

$\frac{1}{\cos X \sin X} = \frac{1}{\cos X \sin X}$ ✓

5) $\csc X - 1 = \frac{\cot^2 X}{\csc X + 1}$

$= \frac{\csc^2 X - 1}{\csc X + 1}$

$= \frac{(\csc X + 1)(\csc X - 1)}{\csc X + 1}$

$\csc X - 1 = \csc X - 1$ ✓

HW: Pg 434

16, 19, 20

Day 3 Verifying Trig Identities (Day 2)

1) $\dfrac{(\tan\theta + \cot\theta)}{\sec^2\theta} = \cot\theta$

$\dfrac{\sin\theta}{\sin\theta}\left(\dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta}\right)\dfrac{\cos\theta}{\cos\theta} =$

$\dfrac{1}{\cos^2\theta}$

$\left(\dfrac{\sin^2\theta}{\sin\theta\cos\theta} + \dfrac{\cos^2\theta}{\sin\theta\cos\theta}\right) =$

$\dfrac{1}{\cos^2\theta}$

$\dfrac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta} =$

$\dfrac{1}{\cos^2\theta}$

$\dfrac{1}{\sin\theta\cos\theta} \cdot \dfrac{\cos^2\theta}{1} =$

$\dfrac{\cos\theta}{\sin\theta} =$

$\cot\theta = \cot\theta$ ✓

2) $\frac{\cos\theta}{1-\sin\theta} = \frac{1+\sin\theta}{\cos\theta}$

$\frac{\cos\theta}{1-\sin\theta} \cdot \frac{1+\sin\theta}{1+\sin\theta} =$

$\frac{\cos\theta + \cos\theta\sin\theta}{1-\sin^2\theta} =$

$\frac{\cos\theta(1+\sin\theta)}{\cos^2\theta} =$

$\frac{1+\sin\theta}{\cos\theta} = \frac{1+\sin\theta}{\cos\theta}$

3) $(\csc x - \cot x)^2 = \frac{1-\cos x}{1+\cos x}$

$(\csc x + \cot x)(\csc x - \cot x) =$

$\csc^2 x - 2\csc x \cot x + \cot^2 x =$

$\frac{1}{\sin^2 x} - 2\frac{1}{\sin x}\cdot\frac{\cos x}{\sin x} + \frac{\cos^2 x}{\sin^2 x}$

$\frac{1-2\cos x + \cos^2 x}{\sin^2 x}$

$\frac{(1-\cos x)^2}{1-\cos^2 x}$

$\frac{(1-\cos x)(1-\cos x)}{(1-\cos x)(1+\cos x)}$

$\frac{1-\cos x}{1+\cos x} = \frac{1-\cos x}{1+\cos x}$ ✓

HW: Pg 434 # 27, 28

Unit 6 Day #4 – Sum & Difference Identities

Objective – You will use the sum and difference identities for the sine, cosine and tangent functions.

Functions of the Sum of Two Angles

$\sin(A+B) = \sin A \cos B + \cos A \sin B$

$\cos(A+B) = \cos A \cos B - \sin A \sin B$

$\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

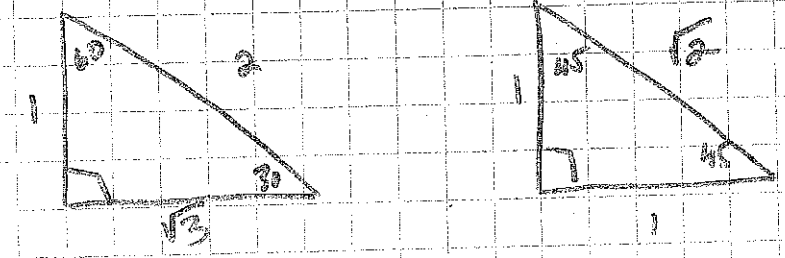
Functions of the Difference of Two Angles

$\sin(A-B) = \sin A \cos B - \cos A \sin B$

$\cos(A-B) = \cos A \cos B + \sin A \sin B$

$\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$

Remember These???



1) Evaluate: $\sin(x-90)^\circ$

$\sin(x+90) = \sin x \cos 90 - \cos x \sin 90$

$= \sin x (0) - \cos x \cdot 1$

$= 0 - \cos x$

$= -\cos x$

2) Evaluate: $\cos(x+270)^\circ$

$\cos(x+270) = \cos x \cos 270 - \sin x \cdot \sin 270$

$= \cos x (0) - \sin x (-1)$

$= 0 + \sin x$

$= \sin x$

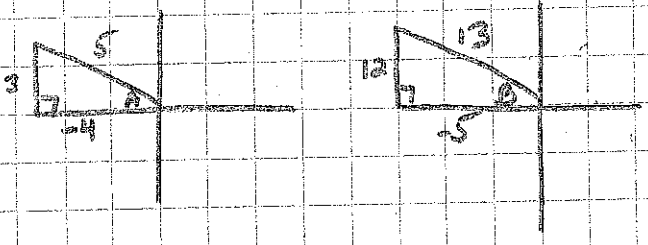
3) Evaluate: $\tan(45^\circ - x)$

$$\tan(45-x) = \frac{\tan 45 - \tan x}{1 + \tan 45 \cdot \tan x}$$

$$= \frac{1 - \tan x}{1 + 1 \cdot \tan x}$$

$$= \boxed{\frac{1-\tan x}{1+\tan x}}$$

4) A & B are in Quadrant II, $\sin A = \frac{3}{5}$ and $\sin B = \frac{12}{13}$.
Evaluate $\cos(A-B)$



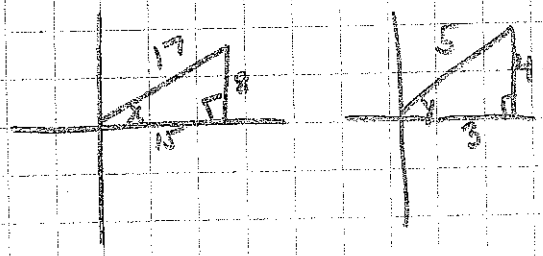
$$\cos(A-B) = \cos A \cos B + \sin A \sin B$$

$$= \frac{-4}{5} \cdot \frac{-5}{13} + \frac{3}{5} \cdot \frac{12}{13}$$

$$= \frac{20}{65} + \frac{36}{65}$$

$$= \boxed{\frac{56}{65}}$$

5) x and y are in Quadrant I, $\sin x = \frac{8}{17}$ and $\cos y = \frac{3}{5}$.
Evaluate $\tan(x-y)$



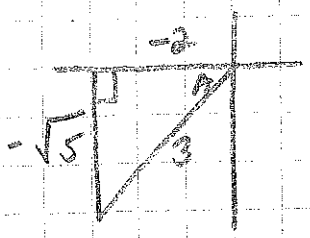
$$\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \cdot \tan y}$$

$$= \frac{\frac{8}{15} - \frac{4}{3}}{1 + \frac{8}{15} \cdot \frac{4}{3}}$$

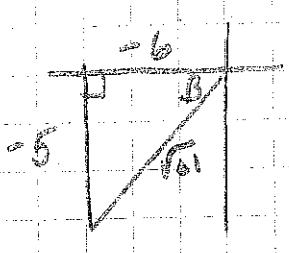
$$= \frac{\frac{8}{15} - \frac{20}{15}}{1 + \frac{32}{45}}$$

$$= \frac{\frac{-12}{15}}{\frac{77}{45}} = \frac{-12}{15} \cdot \frac{3}{77} = \boxed{\frac{-36}{77}}$$

6) A+B are in Quadrant III, $\sec A = -\frac{3}{2}$ ($\frac{H}{A}$) and $\tan B = \frac{5}{6}$.
Evaluate $\sin(A+B)$



$(-2)^2 + x^2 = 3^2$
$x^2 + 4 = 9$
$x^2 = 5$



$5^2 + 6^2 = x^2$
$25 + 36 = x^2$
$61 = x^2$
$x = \sqrt{61}$

$\sin(A+B) = \sin A \cos B + \cos A \sin B$

$= \frac{-\sqrt{5}}{3} \cdot \frac{-6}{\sqrt{61}} + \frac{-2}{3} \cdot \frac{-5}{\sqrt{61}}$

$= \frac{6\sqrt{5}}{3\sqrt{61}} + \frac{10}{3\sqrt{61}}$

$= \boxed{\frac{10 + 6\sqrt{5}}{3\sqrt{61}}}$

Homework:

1) Evaluate: $\cos(x-180)$

2) Evaluate $\sin(180+x)$

3) If x and y are in Quadrant I, $\sin x = \frac{5}{13}$ and $\tan y = \frac{4}{3}$, find $\sin(x-y)$

4) A+B are in Quadrant II, $\cos A = -\frac{8}{17}$ and $\tan B = -\frac{1}{3}$
Evaluate $\cos(A+B)$

Unit 6 Day #5 – Sum & Difference Identities (Day #2)

Same Intro as Previous Day

Examples

1) Find the exact value of $\sin 105°$

$\sin 105 = \sin(45 + 60)$

$= \sin 45 \cos 60 + \cos 45 \cdot \sin 60$

$= \frac{1}{\sqrt{2}} \cdot \frac{1}{2} + \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2}$

$= \frac{1}{2\sqrt{2}} + \frac{\sqrt{3}}{2\sqrt{2}}$

$= \boxed{\frac{1+\sqrt{3}}{2\sqrt{2}}}$

2) Find the exact value of $\cos 195°$

$\cos 195 = (45 + 150)$

$= \cos(45 + 30) = \cos 45 \cos 30 - \sin 45 \sin 30$

$= \frac{1}{\sqrt{2}} \cdot \frac{-\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \cdot \frac{1}{2}$

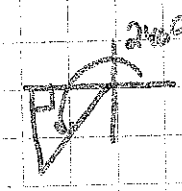
$= \frac{-\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}} = \frac{-1-\sqrt{3}}{2\sqrt{2}}$

3) Find the exact value of $\tan 285°$

$\tan 285° = (45 + 240)$

$= (45 + 60)$

$240 - 180 = 60$



$\tan(45 + 60) = \frac{\tan 45 + \tan 60}{1 - \tan 45 \tan 60}$

$= \frac{1 + \sqrt{3}}{1 - (1 \cdot \sqrt{3})} = \frac{1+\sqrt{3}}{1-\sqrt{3}} \cdot \frac{1+\sqrt{3}}{1+\sqrt{3}}$

$= \frac{1 + 2\sqrt{3} + 3}{1 - 3} = \frac{4 + 2\sqrt{3}}{-2}$

$= -2 - \sqrt{3}$

4) Verify: $\sec(A-B) = \dfrac{\sec A \sec B}{1+\tan A \tan B}$

$$\frac{\frac{1}{\cos A}\cdot\frac{1}{\cos B}}{1+\frac{\sin A}{\cos A}\cdot\frac{\sin B}{\cos B}} \qquad \frac{\cos A \cos B}{\cos A \cos B}$$

$$\frac{1}{\cos A\cos B + \sin A \sin B}$$

$$\frac{1}{\cos(A-B)}$$

$$\sec(A-B) = \sec(A-B)$$

Homework

1) Find the exact value of $\cos 15°$

2) Find the exact value of $\tan 165°$

3) Verify: $\sin(A+B) = \dfrac{\tan A + \tan B}{\sec A \sec B}$

Unit 6 Day # 6 - Double and Half Angle Identities

Objective - You will use the double and half and Identities for the sine, cosine and tangent functions.

Functions of the Double Angle

$\sin 2A = 2\sin A \cos A$

$\cos 2A = \cos^2 A - \sin^2 A$

$\cos 2A = 2\cos^2 A - 1$

$\cos 2A = 1 - 2\sin^2 A$

$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$

Functions of the Half Angle

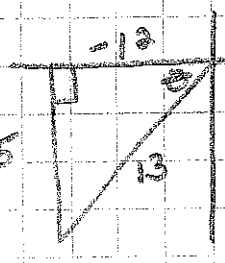
$\sin \frac{1}{2}A = \pm\sqrt{\frac{1-\cos A}{2}}$

$\cos \frac{1}{2}A = \pm\sqrt{\frac{1+\cos A}{2}}$

$\tan \frac{1}{2}A = \pm\sqrt{\frac{1-\cos A}{1+\cos A}}$

Examples:

1) If $\tan\theta = \frac{5}{12}$ and θ is in Quad III, find $\sin 2\theta$, $\cos 2\theta$, $\tan 2\theta$ and $\cos\frac{\theta}{2}$



- $\sin 2\theta = 2\sin\theta\cos\theta$
  $= 2\left(\frac{-5}{13}\right)\left(\frac{-12}{13}\right)$
  $= \boxed{\frac{120}{169}}$

- $\cos 2\theta = \cos^2\theta - \sin^2\theta$
  $= \left(\frac{-12}{13}\right)^2 - \left(\frac{-5}{13}\right)^2$
  $= \frac{144}{169} - \frac{25}{169} = \boxed{\frac{119}{169}}$

$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta} = \frac{2\cdot(5/12)}{1-(5/12)^2}$

$= \frac{10/12}{1-\frac{25}{144}} = \frac{\frac{10}{12}}{\frac{144}{144}-\frac{25}{144}} = \frac{\frac{10}{12}}{\frac{119}{144}}$

$= \frac{10}{12}\cdot\frac{144}{119}$

$= \boxed{\frac{120}{119}}$

$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$

$= \pm\sqrt{\frac{1+\frac{-12}{13}}{2}}$

$= \pm\sqrt{\frac{\frac{13}{13}-\frac{12}{13}}{2}}$

$= \pm\sqrt{\frac{\frac{1}{13}}{2}}$

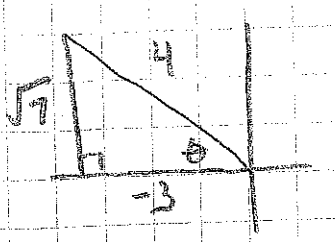
$= \pm\sqrt{\frac{1}{13}\cdot\frac{1}{2}}$

$= \pm\sqrt{\frac{1}{26}}$

$=$

II

2) If $\sec\theta = -\frac{4}{3}$ and $90° < \theta < 180°$, find $\sin 2\theta$, $\cos 2\theta$, $\tan 2\theta$ and $\sin\frac{\theta}{2}$.



$(-3)^2 + b^2 = 4^2$

$b^2 + 9 = 16$

$-9 \quad -9$

$\sqrt{b^2} = \sqrt{7}$

$b = \sqrt{7}$

$\sin 2\theta = 2\sin\theta\cos\theta$

$= 2\left(\frac{\sqrt{7}}{4}\right)\left(-\frac{3}{4}\right)$

$= \boxed{-\frac{6\sqrt{7}}{16}}$

$\cos 2\theta = 2\cos^2\theta - 1$

$= 2\left(-\frac{3}{4}\right)^2 - 1$

$= 2\left(\frac{9}{16}\right) - 1$

$= \frac{18}{16} - \frac{16}{16}$

$= \frac{2}{16} = \boxed{\frac{1}{8}}$

$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$

$= \frac{2\left(\frac{\sqrt{7}}{-3}\right)}{1 - \left(\frac{\sqrt{7}}{-3}\right)^2} = \frac{-\frac{2\sqrt{7}}{3}}{\frac{9}{9} - \frac{7}{9}} = -\frac{2\sqrt{7}}{3} \div \frac{2}{9}$

$= -\frac{2\sqrt{7}}{3} \cdot \frac{9}{2}$

$= \boxed{-3\sqrt{7}}$

$\sin\frac{1}{2}\theta = \pm\sqrt{\frac{1-\cos\theta}{2}}$

$= \pm\sqrt{\frac{1-\left(-\frac{3}{4}\right)}{2}}$

$= \pm\sqrt{\frac{\frac{4}{4} + \frac{3}{4}}{2}}$

$= \pm\sqrt{\frac{7}{4} \cdot \frac{2}{1}}$

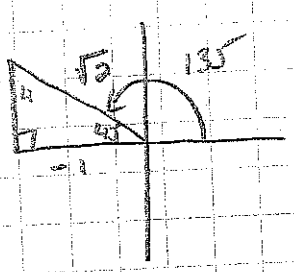
$= \pm\sqrt{\frac{7}{2}}$

3) Find the exact value of $\sin 67.5$

$67.5 \times 2 = 135^\circ$

135° in Quad II so +.

$$\sin\frac{135}{2} = \pm\sqrt{\frac{1-\cos 135^\circ}{2}}$$



$$= +\sqrt{\frac{1-\left(-\frac{1}{\sqrt{2}}\right)}{2}}$$

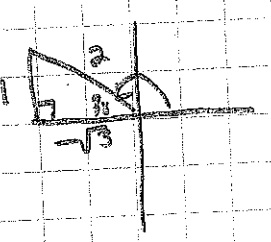
$$= +\sqrt{\frac{1-\left(-\frac{\sqrt{2}}{2}\right)}{2}}$$

$$= +\sqrt{\frac{\frac{2}{2}+\frac{\sqrt{2}}{2}}{2}} = +\sqrt{\frac{2+\sqrt{2}}{2}\cdot\frac{2}{4}}$$

$$= \sqrt{2+\sqrt{2}}$$

4) Find the exact value of $\tan\frac{5\pi}{12}$.

$\therefore \tan\frac{5\pi}{12} = \tan 75 = \pm\tan\frac{150}{2}$



$$\tan\frac{150}{2} = \pm\sqrt{\frac{1-\cos 150^\circ}{1+\cos 150^\circ}}$$

$$= -\sqrt{\frac{\left(1-\left(-\frac{\sqrt{3}}{2}\right)\right)^2}{\left(1+\left(-\frac{\sqrt{3}}{2}\right)\right)^2}}$$

$$= -\sqrt{\frac{2+\sqrt{3}}{2-\sqrt{3}}\cdot\frac{2+\sqrt{3}}{2+\sqrt{3}}} = -\sqrt{\frac{4+4\sqrt{3}+3}{4-3}}$$

$$= -\sqrt{\frac{7+4\sqrt{3}}{1}}$$

$$= -\sqrt{7+4\sqrt{3}}$$

Homework:

1) If $\cot\theta = -\frac{8}{15}$ and $90° < \theta < 180°$, find $\sin 2\theta$, $\cos 2\theta$, $\tan 2\theta$ and $\cos\frac{\theta}{2}$

2) If $\csc\theta = -\frac{5}{3}$ and $270° < \theta < 360°$, find $\sin 2\theta$, $\cos 2\theta$, $\tan 2\theta$ and $\sin\frac{\theta}{2}$

3) Find the exact value of $\cos\frac{7\pi}{12}$.

Unit 6 Day #7 - Double and Half Angle Identities (Day #2)

Example #1

Verify: $\csc 2\theta \stackrel{?}{=} \frac{1}{2}\sec\theta\csc\theta$

$\frac{1}{\sin 2\theta} \stackrel{?}{=} \frac{1}{2}\cdot\frac{1}{\cos\theta}\cdot\frac{1}{\sin\theta}$

$\frac{1}{2\sin\theta\cos\theta} \stackrel{?}{=} \frac{1}{2\sin\theta\cos\theta}$ ✓

2) Verify: $\cos A - \sin A \stackrel{?}{=} \frac{\cos 2A}{\cos A + \sin A}$

$\stackrel{?}{=} \frac{\cos^2 A - \sin^2 A}{\cos A + \sin A}$

$\cos A - \sin A \stackrel{?}{=} \cos A - \sin A$

3) Verify: $(\sin\theta + \cos\theta)^2 - 1 \stackrel{?}{=} \sin 2\theta$

$(\sin\theta + \cos\theta)(\sin\theta + \cos\theta) - 1 \stackrel{?}{=} 2\sin\theta\cos\theta$

$\sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta - 1 \stackrel{?}{=}$

$2\sin\theta\cos\theta + 1 - 1 \stackrel{?}{=}$

$2\sin\theta\cos\theta \stackrel{?}{=} 2\sin\theta\cos\theta$ ✓

4) Verify: $\cos x - 1 = \frac{\cos 2x - 1}{2(\cos x + 1)}$

$= \frac{2\cos^2 x - 1 - 1}{2\cos x + 2}$

$= \frac{2\cos^2 x - 2}{2\cos x + 2}$

$= \frac{2(\cos^2 x - 1)}{2(\cos x + 1)}$

$\cos x - 1 = \cos x - 1$ ✓

Unit 6 Day #7 – Solving Trig. Equations

Objectives: You will solve trig. equations.

Solving Trig Equations

1) Rewrite in terms of one trig function if possible using the identities.
2) Solve for the trig function by factoring
3) Solve for the angle.
4) Find all possible solutions using reference angles.

Examples

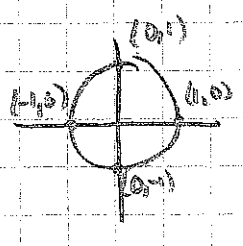
1) Solve $\sin x \cos x - \frac{1}{2}\cos x = 0$ for all values of x in the interval $0° \le x \le 360°$

$\sin x \cos x - \frac{1}{2}\cos x = 0$

$\cos x(\sin x - \frac{1}{2}) = 0$

$\cos x = 0$ $\quad$ $\sin x - 1/2 = 0$

$\sin x = 1/2$



$x = 90$

$x = 270$

$x = 30$

II: $180 - 30$

$x = 150°$

$x = 30, 90, 150, 270$

2) Solve for $4\sin^2 x + 1 = -4\sin x$ for all values of x in the interval $0 \leq x < 360$

$$4\sin^2 x + 1 = -4\sin x$$

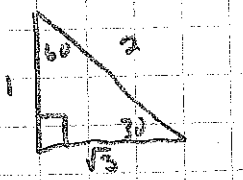
$$4\sin^2 x + 4\sin x + 1 = 0$$

$$(2\sin x + 1)(2\sin x + 1) = 0$$

$$2\sin x + 1 = 0$$

$$\frac{2\sin x}{2} = \frac{-1}{2}$$

$$\sin x = -1/2$$



III: $x = 180 + 30$

$x = 210$

$x = 30$

IV: $360 - 30$

$x = 330$

$x = 210, 330$

3) Solve $\cos^2 x - \cos x + 1 = \sin^2 x$ for all values of x in the interval $0° \leq x < 360$

$$\cos^2 x - \cos x + 1 = \sin^2 x$$

$$\cos^2 x - \cos x + 1 = 1 - \cos^2 x$$

$$+\cos^2 x \quad -1 \quad -1 + \cos^2 x$$

$$2\cos^2 x - \cos x = 0$$

$$\cos x(2\cos x - 1) = 0$$

$\cos x = 0$

$x = 90$

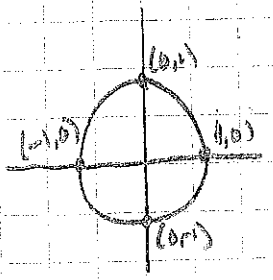
$x = 270$

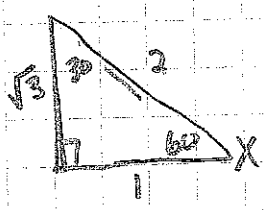
$2\cos x - 1 = 0$

$\frac{2\cos x}{2} = \frac{1}{2}$

$\cos x = 1/2$ +

$x = 60°$





I, IV:

$360 - 60 = 300°$

$x = 60°, 90, 270, 300°$

4) Solve $2\sin x + \csc x = 3$ for all values of x in the interval $0 \leq x < 360$

$$2\sin x + \csc x = 3$$

$$\sin x\left(2\sin x + \frac{1}{\sin x} - 3 = 0\right)$$

$$2\sin^2 x + 1 - 3\sin x = 0$$

$$2\sin^2 x - 3\sin x + 1 = 0$$

$$(2\sin x - 1)(\sin x - 1) = 0$$

$2\sin x - 1 = 0$

$2\sin x = 1$

$\frac{2\sin x}{2} = \frac{1}{2}$

$\sin x = 1/2$

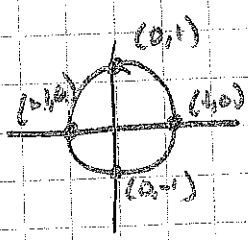
$x = 30$

II. $x = 180 - 30 = 150$

$\sin x - 1 = 0$

$\sin x = 1$

$x = 90$



$x = 30, 90, 150$

Homework:

1) Solve $\tan^2 x + 2\tan x + 1 = 0$ for all values of x in the interval $0° \leq x < 360°$.

2) Solve $2\cos^2 x = 3\sin x$ for all values of x in the interval $0° \leq x < 360°$

Unit 6 Day 9 → Solving Trig. Equations (Day #2)

1) $-1-3\sin x = \cos 2x$ for all values of x in the interval $0° \le x < 360°$

$-1-3\sin x = \cos 2x$

$-1-3\sin x = 1-2\sin^2 x$

$+2\sin^2 x \quad -1 \quad -1+2\sin^2 x$

$2\sin^2 x - 3\sin x - 2 = 0$

$(2\sin x + 1)(\sin x - 2) = 0$

$2\sin x + 1 = 0$ | $\sin x - 2 = 0$

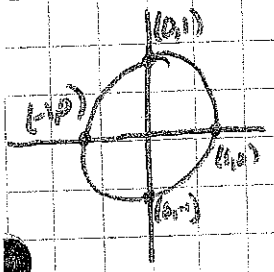
$\frac{2\sin x}{2} = \frac{-1}{2}$ | $\sin x = 2$ Error

$\sin x = -\frac{1}{2}$

$x = 30°$

III: $180 + 30 = \boxed{210°}$

IV: $360 - 30 = \boxed{330°}$



2) Solve $\sec^2 x + 2\sec x = 0$ for all values of x in the interval $0° \le x < 360°$

$\sec^2 x + 2\sec x = 0$

$\sec x(\sec x + 2) = 0$

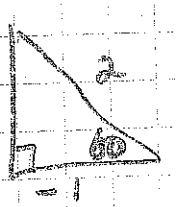
$\sec x = 0$ | $\sec x + 2 = 0$

$\frac{1}{\cos x} = \frac{0}{1}$ | $\frac{1}{\cos x} = \frac{-2}{1}$

$\cos x(0) = 1$ | $-2\cos x = 1$

$0 \neq 1$ Error | $\frac{-2\cos x}{-2} = \frac{1}{-2}$

$\cos x = -1/2$ — II, III



$x = 60$

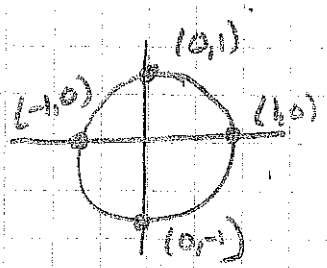
II: $180 - 60 = \boxed{120°}$

IV: $180 + 60 = \boxed{240°}$

3) $\sin 2x + \sin x = 0$

$2\sin x \cos x + \sin x = 0$

$\sin x(2\cos x + 1) = 0$



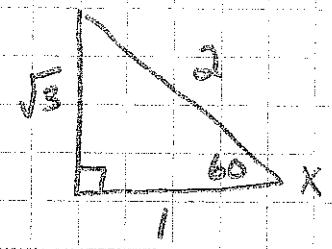
$\sin x = 0$

$x = 0, 180$

$2\cos x + 1 = 0$

$\frac{2\cos x}{2} = \frac{-1}{2}$

$\cos x = -\frac{1}{2}$ II, III



$x = 60$

II: $180 - 60 = \boxed{120^\circ}$

III $180 + 60 = \boxed{240^\circ}$

$\boxed{x = 0, 120, 180, 240}$

4) $\cos 2x = \cos^2 x - \cos x + 1$ over $0 \le x < 360$

$2\cos^2 x - 1 = \cos^2 x - \cos x + 1$

$-\cos^2 x \quad +1 \quad -\cos^2 x \quad +\cos x \quad -1$

$\cos^2 x + \cos x - 2 = 0$

$(\cos x - 1)(\cos x + 2) = 0$

$\cos x - 1 = 0$

$\cos x = 1$

$\boxed{x = 0^\circ}$

$\cos x + 2 = 0$

$\cos x = -2$ Error

HW:

1) Solve $\sin 2x + \sin x = 0$ for all values of x in the interval $0 \le x < 360$

2) Solve $\cos^2 x = 2 - \cos x$ for all values of x in the interval $0 \le x < 360^\circ$

5) Solve $\sin 2x - 1 = 0$ for all values of x in the interval $0° \leq x \leq 360$

$\sin 2x - 1 = 0$

$2\sin x \cos x - 1 = 0$ Uh-Oh!

$\sin 2x = 1$

$\boxed{x = 45°}$

$\sin x = 1$

$\sin 90 = 1$